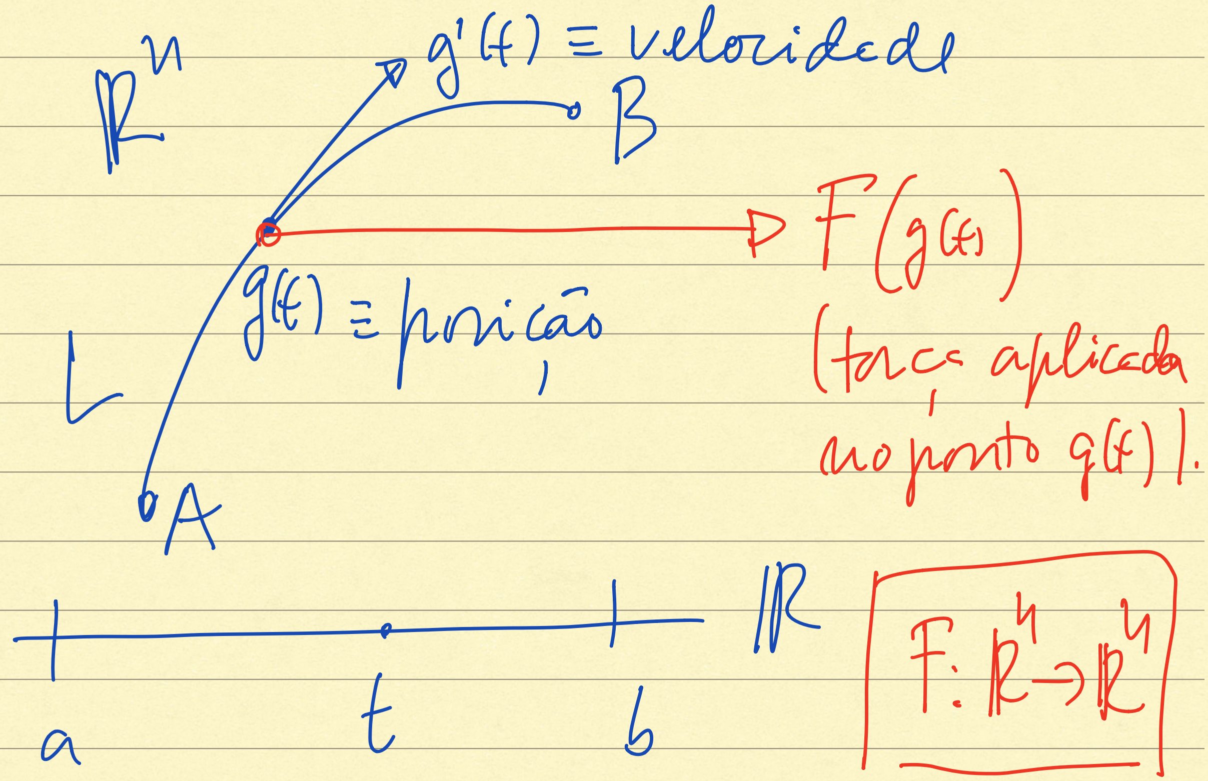
Integral de linha de um campo Vectorial (Trabalho realizado por um campo de forças)



$$W \equiv \int_L F = \int_a^b \underbrace{F(g(t))}_{\text{força}} \cdot \underbrace{g'(t)}_{\text{velocidade}} \, \underbrace{dt}_{\text{tempo}}$$

$$W = \int_L F = \boxed{\int_L F \cdot dg} = \int_a^b F(g(t)) \cdot \underbrace{g'(t)\,dt}_{dg}$$

$$= E_c(B) - E_c(A) = \Delta E_c$$

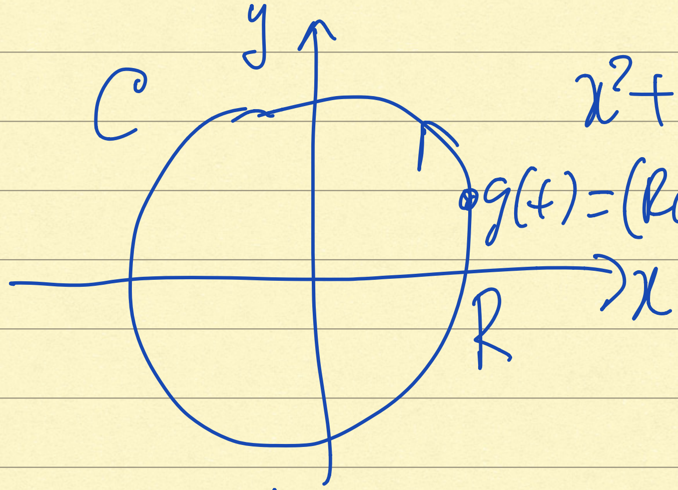
$$E_c(x) = \frac{1}{2} m \|v_x\|^2$$

Variação de energia cinética

||

Exemplo: $F(x,y) = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$

$(x,y) \neq (0,0)$



$x^2+y^2=R^2$

$g(t)=(R\cos t, R\operatorname{sen} t)$

$0\leq t\leq 2\pi$

1 volta

$$\int_C F = 2\pi! \quad \forall R>0$$

$$\| $$

2) $F(x,y)=\left(\dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2}\right)$

$(x,y)\neq(0,0)$

<u>Exercício</u>: $F=\nabla\varphi$

$$\varphi(x,y)=\frac{1}{2}\log(x^2+y^2)+\underline{C}$$

<u>constante</u>

$$\frac{\partial \varphi}{\partial x} = \frac{1}{2}\,\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2} \checkmark$$

$$\frac{\partial \varphi}{\partial y} = \frac{1}{2}\,\frac{2y}{x^2+y^2} = \frac{y}{x^2+y^2} \checkmark$$

$$\boxed{F = \nabla \varphi}$$

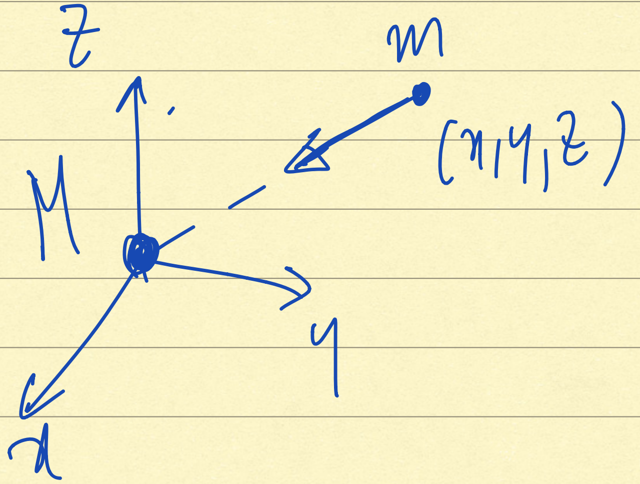
3) $$F(x,y,z) = \frac{1}{(x^2+y^2+z^2)^{3/2}}(x,y,z)$$

$$(x,y,z) \neq (0,0,0)$$

$$F = \nabla \varphi \quad : \varphi(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} + C$$

$$\|F(x,y,z)\| = \frac{\|(x,y,z)\|}{(x^2+y^2+z^2)^{3/2}}$$

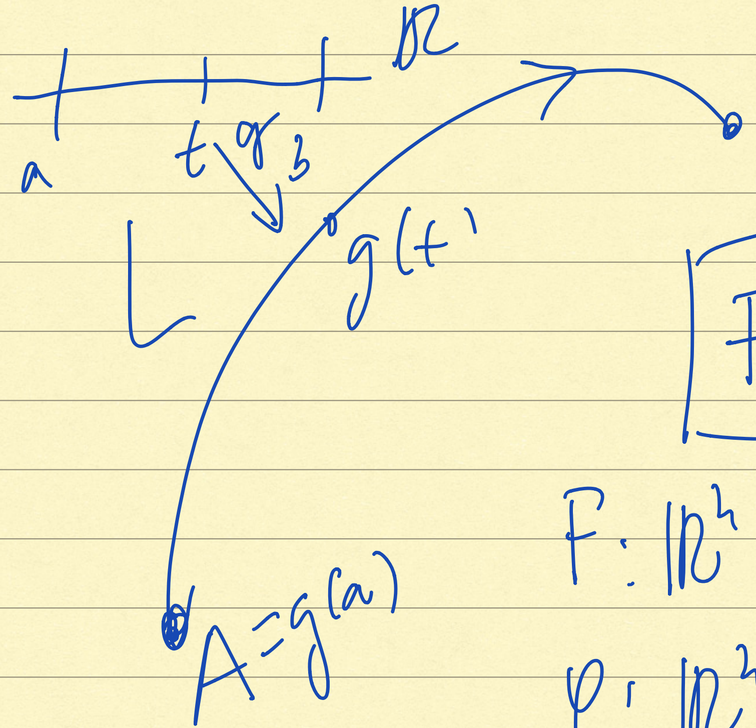
$$= \frac{\|(x,y,z)\|}{\|(x,y,z)\|^3} = \frac{1}{\|(x,y,z)\|^2}$$



$$F : \mathbb{R}^3 \setminus \{(0,0,0)\} \to \mathbb{R}^3$$

$$\varphi(x,y,z) = \frac{1}{\|(x,y,z)\|} \equiv \boxed{\text{potencial escalar de } F}$$

$$\varphi : \mathbb{R}^3 \setminus \{(0,0,0)\} \longrightarrow \mathbb{R}$$



$$\boxed{F = \nabla \varphi}$$

$F : \mathbb{R}^2 \to \mathbb{R}^2,$

$\varphi : \mathbb{R}^2 \to \mathbb{R}.$

$$W = \int_L F \cdot dg = \int_a^b F(g(t)) \cdot g'(t)\, dt$$

$$= \int_a^b \nabla \varphi (g(t)) \cdot g'(t)\, dt$$

$$= \int_a^b \frac{d}{dt} \varphi(g(t))\, dt$$

$$= \varphi(g(b)) - \varphi(g(a))$$

$$= \varphi(B) - \varphi(A)$$

$$\text{———————— } \| \text{ ————————}$$

Se $F = \nabla \varphi$, então

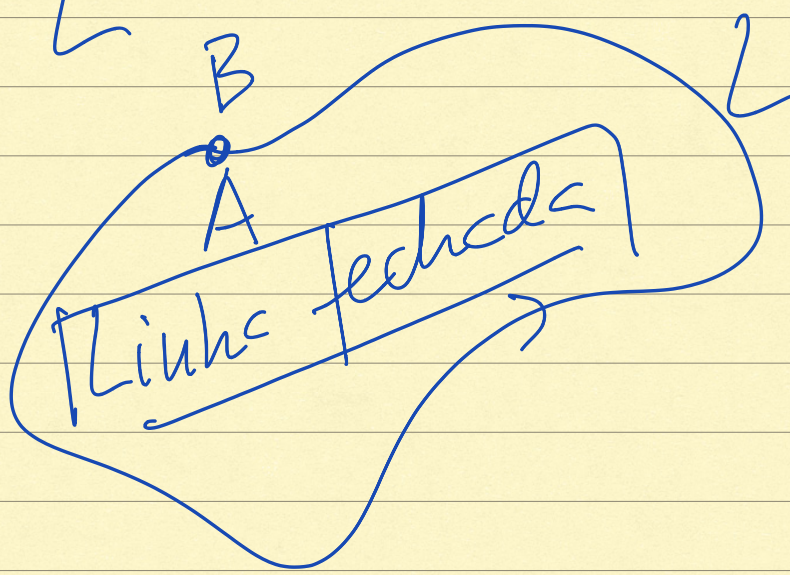
$$\int_L F = \varphi(B) - \varphi(A)$$

teorema fundamental do cálculo em $\mathbb{R}^n$

Consequências: $\boxed{F = \nabla \varphi}$

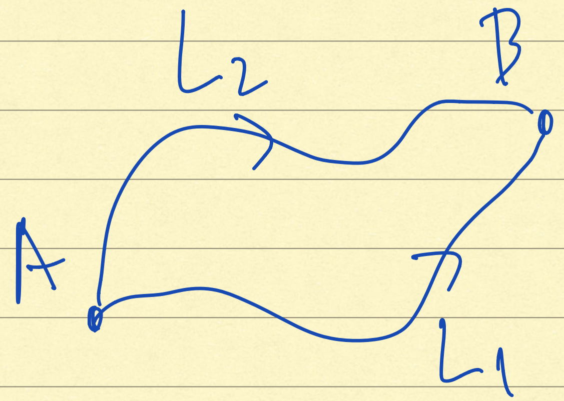
1) Se $A \equiv B$, então

$$\int_L F = 0 \quad !!!$$



2) $\displaystyle\int_L F = \varphi(B) - \varphi(A)$ só depende de A e de B.

$\Rightarrow$ Não depende da linha L.



Identificar campos gradientes.

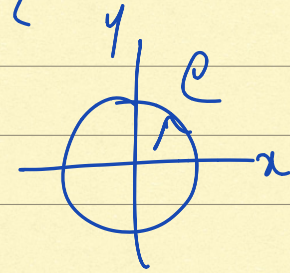
Teorema (a demonstrar mais tarde)

$F: \mathbb{R}^2 \to \mathbb{R}^2$ é gradiente sse

$$\int_L F = 0 \, , \quad \forall_L \text{ fechada.}$$

$$\Longrightarrow F(x,y) = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$$

$F: \mathbb{R}^2 \setminus \{(0,0)\} \longrightarrow \mathbb{R}^2$



$$\int_{C} F = 2\pi \neq 0$$

$C: x^2+y^2 = R^2 \circlearrowleft$

Portanto, F não é gradiente!

Critério para identificar campos gradientes?